151 A Dis. 2A

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+ by appointment
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(or Piazza)

- 151A in subject
- No email on Sundays. 24 hr response (max)

- Notes, recordings, code on Canvas

Today:

- Calculus tools
- MATLAB

Calculus Review:

- Intermediate Value Thm (IVT)
- Taylor's theorem

- IVT: function is continuous
- Taylor: $C^n [a,b]$

Ex/ Taylor polynomial around $\pi/4$ to approx. $\cos(42^\circ)$ to an accuracy of 10^{-6}

Soln (Solution)

Recall:

$$f(x) = P_n(x) + R_n(x)$$

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1}$$

$\xi(x)$ between x, x_0

(\xi)

Idea: Find n that guarantees

center: $\pi/4$

approx: $\cos(42^\circ)$

$$|R_n(x)| < 10^{-6}$$

Step 0

$$42^\circ \Rightarrow 42\left(\frac{\pi}{180}\right) = \frac{7\pi}{30}$$

Step 1 Identify variables

$$x = \frac{7\pi}{30}$$

$$x_0 = \pi/4$$

$$f(x) = \cos(x)$$

Step 2

$$R_n\left(\frac{7\pi}{30}\right) = \frac{f^{(n+1)}\left(\xi\left(\frac{7\pi}{30}\right)\right)\left(\frac{7\pi}{30} - \frac{\pi}{4}\right)^{n+1}}{(n+1)!}$$

$f(x) = \cos(x) \rightarrow$ all derivs. are $\pm\sin(x)$, $\pm\cos(x)$

& all bounded by 1 (absolute value)

$$\left|R_n\left(\frac{7\pi}{30}\right)\right| \leq \frac{1}{(n+1)!}\left|\frac{7\pi}{30} - \frac{\pi}{4}\right|^{n+1} < 10^{-6}$$

$n=3$

$$P_n\left(\frac{7\pi}{30}\right) = \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\left(\frac{7\pi}{30}-\frac{\pi}{4}\right)$$

$$- \frac{\cos(\pi/4)}{2}\left(\frac{7\pi}{30}-\frac{\pi}{4}\right)^2$$

$$+ \frac{\sin(\pi/4)}{3!}\left(\frac{7\pi}{30}-\frac{\pi}{4}\right)^3$$

IVT ex Show that:

$$(x-2)^2 - \ln(x) = 0$$

has at least 1 solution in the intervals:

$$[1,2], \quad [e,4]$$

Soln IVT

Let $f(x) = (x-2)^2 - \ln(x)$

$(x-2)^2$ polynomial, $\ln(x)$ cont. on each interval, so $f(x)$ is cont. on each interval.

$f(1) = (1-2)^2 - \ln(1) > 0$

$f(2) = (2-2)^2 - \ln(2) < 0$

$\exists$ "there exists"

By IVT, $\exists \; x^* \in (1, 2)$ such that $f(x^*) = 0$.