Announcement:

- Office hour: Tuesday 1:30-2:30 PM (Zoom)

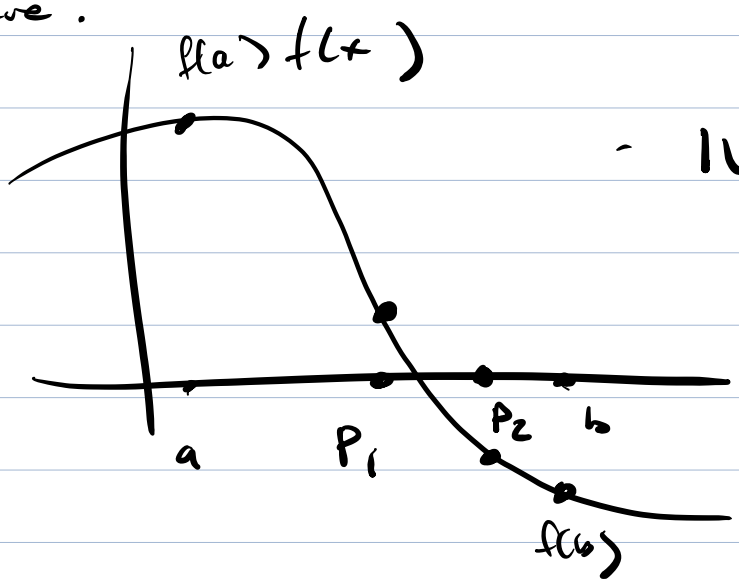
Today:

- Bisection Method
- Intro to fixed point problems

HW questions?

Bisection Method

Picture:



- IVT

Ex/ $f(x) = x^2 - 2$

4 iterations of B.M. on $[1,2]$

Soln **Step 0** • $f(x)$ is continuous (polynomial)

• $f(1) = -1 < 0$

• $f(2) = 2 > 0$ ✓

k	a_k	b_k	p_k	$f(a_k)$	$f(b_k)$	$f(p_k)$
1	1	2	3/2	-1	2	1/4
2	1	3/2	5/4	-1	1/4	-7/16
3	5/4	3/2	11/8	-7/16	1/4	-7/64
4	11/8	3/2	23/16			

Error bound

$$|p_n - p| \leq \frac{b-a}{2^n}, \quad n \geq 1$$

Ex/ $x^3 + x - 4 = 0$

B.M. on $[1,4]$

# of iterations required for an accuracy of 10^{-3}

Soln Step 0 (note): $f(x) = x^3 + x - 4$

is cont. (polynomial)

$f(1) < 0$

$f(4) > 0$ ✓

$$|p_n - p| \leq \frac{b-a}{2^n} < 10^{-3}$$

$$\frac{3}{2^n} < 10^{-3}$$

$$n > 11.6$$

$\Rightarrow$ need no more than 12 iterations

Fixed point formulations

Ex $f(x) = x^3 - 2x + 1$

For each $g(x)$, show that $f(p) = 0$ if and only if $g(p) = p$

a) $g(x) = \frac{1}{2}(x^3 + 1)$

b) $g(x) = \frac{2}{x} - \frac{1}{x^2}$

c) $g(x) = \sqrt{2 - \frac{1}{x}}$

d) $g(x) = -\left(\sqrt[3]{1 - 2x}\right)$

Soln

a) $f(p) = 0 \iff p^3 - 2p + 1 = 0$

$\iff 2p = p^3 + 1$

$\iff p = \frac{1}{2}(p^3+1)$

$g(p) = p$

b) $f(p) = 0 \iff p^3 - 2p + 1 = 0$

$\iff p^3 = 2p - 1$

$p = \frac{2}{p} - \frac{1}{p^2}$ (OK since $f(p)=0$, $p \neq 0$)

$\iff g(p) = p$

c), d) similar