Announcement:

- OH today from 3:30-4:30 PM (Same Zoom link)

Today:

- Comments on HW 2 (+ questions)
- Newton's method
- Secant method
- Fixed point problem
- Matlab

HW 2

3. a) expression $L(x)$
   b) find root of $L(x)$

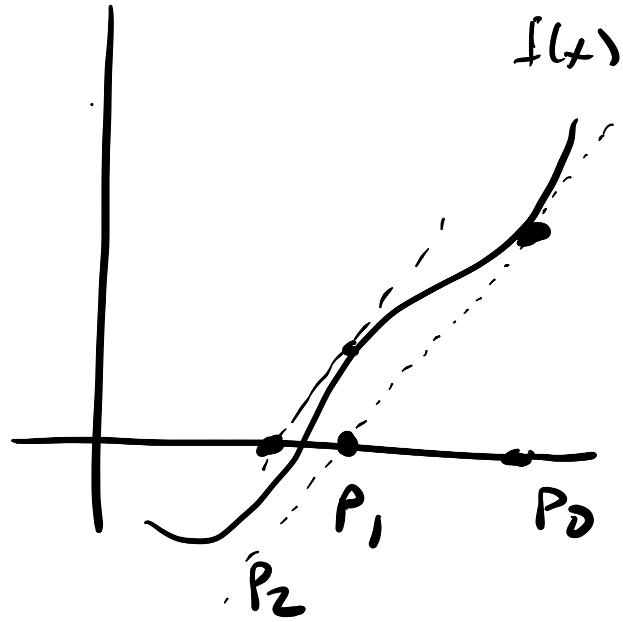
4. Hint: proof of convergence order thm - for FPI (Lecture 7 notes)
   - think Taylor / MVT

5. Residual: $|f(p_n)| < 10^{-5}$

6. $e_{n+1} = \lambda e_n^{\alpha}$ (for any large n)

Newton's Method

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$



Check: $f'(p_n) \neq 0$

Ex $f(x) = e^x - x^2$

$p_0 = -1$

Use N.M. to find p_2

Soln $f'(x) = e^x - 2x$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = -1 - \frac{e^{-1} - 1}{e^{-1} + 2}$$

$$\approx -0.7330$$

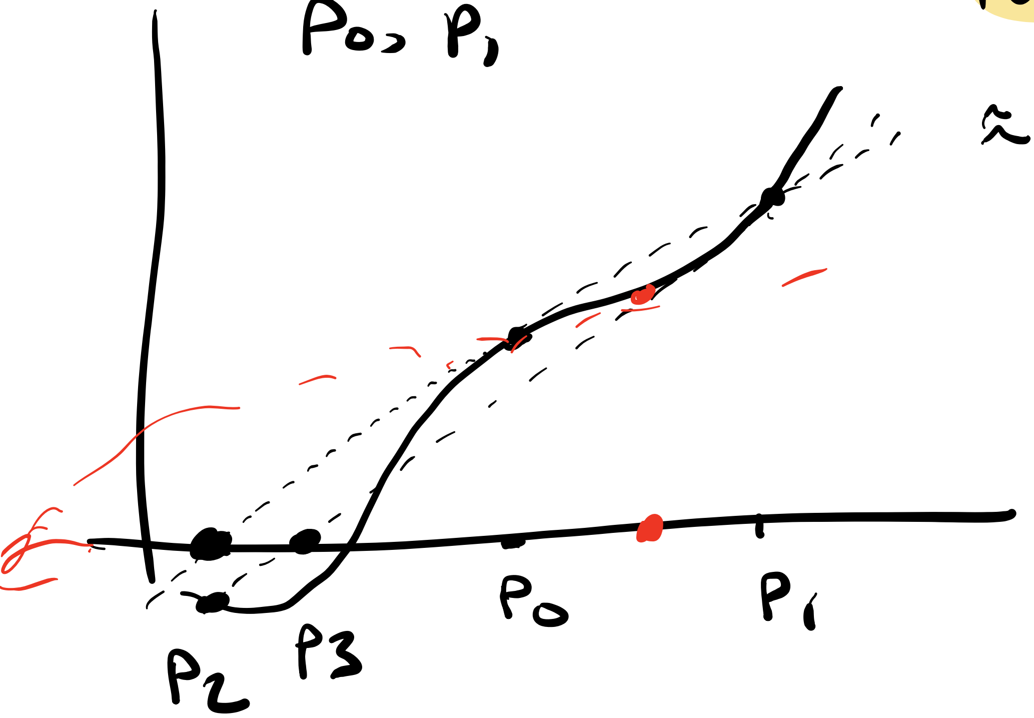
$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \approx -0.7038$$

Secant Method

$$p_{n+1} = p_n - f(p_n)\left(\frac{p_n - p_{n-1}}{f(p_n) - f(p_{n-1})}\right)$$

→ initial points needed: p_0, p_1

$$\approx \frac{1}{f'(p_n)}$$



Ex $f(x) = e^x - x^2$

$p_0 = -1, \quad p_1 = 0$

Use Secant Method to find p_3

Soln $$p_2 = p_1 - f(p_1)\left(\frac{p_1 - p_0}{f(p_1) - f(p_0)}\right)$$

$$= -0.6127$$

$$p_3 = p_2 - f(p_2)\left(\frac{p_2 - p_1}{f(p_2) - f(p_1)}\right)$$

$$= -0.7351$$

Fixed Points

Ex $g(x) = \pi + \frac{1}{2}\sin(x/2)$

Show $\exists$ unique F.P. for $g(x)$ on the interval $[0, 2\pi]$

Soln 1) WTS: $\forall x \in [0, 2\pi]$, $g(x) \in [0, 2\pi]$ (existence of F.P.)

$$-\frac{1}{2} < \frac{1}{2}\sin(x/2) < \frac{1}{2}$$

$$0 < \pi - \frac{1}{2} < \pi + \frac{1}{2}\sin(\frac{x}{2}) < \pi + \frac{1}{2} < 2\pi$$

$\therefore \forall x \in [0, 2\pi]$, $g(x) \in [0, 2\pi]$

So $\exists$ atleast one F.P. for $g(x)$ in $[0, 2\pi]$

2) WTS: $\exists k \in (0,1)$ s.t. $\forall x, y \in [0, 2\pi]$,

$|g(x) - g(y)| \leq k|x - y|$

(uniqueness)

$\forall x, y \in [0, 2\pi]$ $(x \neq y)$, since g is diff., by the MVT $\exists c$ between x, y s.t.

$$\frac{g(y) - g(x)}{y - x} = g'(c)$$

so $$\frac{|g(y)-g(x)|}{|y-x|} = |g'(c)|$$

$$g'(x) = \frac{1}{4}\cos\left(\frac{x}{2}\right), \text{ so } |g'(x)| \le \frac{1}{4} = k$$

$$\frac{|g(y)-g(x)|}{|y-x|} \le \frac{1}{4}$$

$$|g(y)-g(x)| \le \frac{1}{4}|y-x|$$

$\therefore$ F.P. is unique