**Today:**

- HW questions
- Divided differences
- Runge's phenomenon demonstration
- Midterm review

---

## HW4

#1) a) Use fact that

$$p(x_i) = f(x_i)$$

to generate a $3\times3$ linear system

c) $\det(A) \neq 0$ iff $\exists$ a unique solution to

$$Ax = b$$

for any $b$

#4) Theorem: Given $\{x_i\}_{i=0}^{n}$ distinct, $f \in C^{n+1}([a,b])$,

$$p(x) = \sum_{i=0}^{n} f(x_i) L_i(x)$$

$\forall x \in [a,b]$, $\exists\ \xi(x) \in (a,b)$ s.t.

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{k=0}^{n} (x - x_k)$$

#5) "Guess and check" values of n to find the smallest n that works

---

## Divided differences

- Recursive method (easier to add on additional data points)

- different form compared to Lagrange construction, but by uniqueness, these forms are equivalent

**Notation:**

$$f[x_i] = f(x_i)$$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$$

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$$

$$P(x) = f[x_0] + f[x_0, x_1](x - x_0)$$

$$+ f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$+ \dots$$

Ex/ $x_0 = 0$ $\quad f(x_0) = 1$

$x_1 = 2$ $\quad f(x_1) = -1/2$

$x_2 = 3$ $\quad f(x_2) = 4$

<u>Soln</u> $f[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$= \frac{-1/2 - 1}{2 - 0} = \boxed{-\frac{3}{4}}$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{4 - (-1/2)}{3 - 2} = \boxed{\frac{9}{2}}$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= \frac{9/12 - (-3/4)}{3} = \boxed{\frac{7}{4}}$$

$$P(x) = 1 - \frac{3}{4}(x-0) + \frac{7}{4}(x-0)(x-2)$$

$$= \frac{7}{4}x^2 - \frac{17}{4}x + 1$$

Lagrange form: ("usual" construction)

$$P(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$+ f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= 1 \cdot \frac{(x-2)(x-3)}{(1-2)(1-3)} + \left(-\frac{1}{2}\right)\frac{(x-0)(x-3)}{(2-0)(2-3)}$$

$$+ 4\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$= \frac{1}{6}(x^2-5x+6) + \frac{1}{4}(x^2-3x)$

$+ \frac{4}{3}(x^2-2x)$

$= \frac{7}{4}x^2 - \frac{17}{4}x + 1$

---

## Midterm Review

- Review HW 1-4

General topics:

- Calculus theorems (IVT, EVT, MVT / Taylor's thm)
  verify assumptions — error term
- Root-finding methods
  - Bisection method
  - Newton's method
  - Secant method
  ] draw picture
  → existence of a root (IVT)
- Fixed point method
  - error bound
  - convergence order theorem
  - existence + uniqueness

- formulating fixed-point problems from root-finding problems

- Floating point considerations

- Interpolating polynomials
  - Lagrange construction
  - Divided differences
  - error bounds

- formulating fixed-point problems from root-finding problems

Show that:

$$g(p) = p \quad \text{iff} \quad f(p) = 0$$