Announcement: OH today moved to 3:30-4:30 PM
(same link)

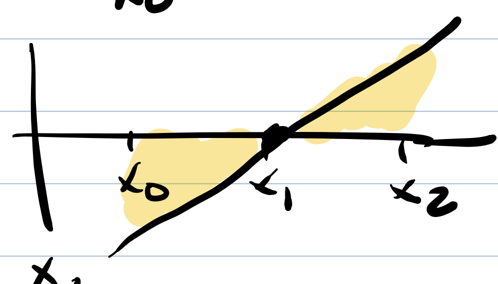
Today:
- HW6 comments
- More on quadrature
- Composite quadrature

HW6

#2b) antisymmetric functions

Given: x_0, x_1, x_2 equispaced

Then: $\int_{x_0}^{x_2} (x - x_1)\, dx = 0$



$\int_{x_0}^{x_2} (x-x_1)^n\, dx = 0$ if n is odd

#3) $\int_1^2 x\ln(x)\,dx$

$$\int u\,dv = uv - \int v\,du$$

$u = \ln(x)$, $dv = x\,dx$

#7) F.T.C. (part II)

$$\frac{d}{dx}\int_0^x g(t)\,dt = g(x)$$

→ no need to modify trap_rule.m
→ use newtonRoot.m (HW2)

→ sendout cubic spline note

Quadrature

Simpson's rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]$$

$$h=\frac{b-a}{2},\quad x_0=a,\ x_1=\frac{a+b}{2},$$
$$x_2=b$$

Ex/ $\int_1^2 x\ln(x)\,dx$ using Simpson's rule

$h=1/2 \quad x_0=1,\ x_1=\frac{3}{2},\ x_2=2$

$$\frac{1/2}{3}\left[f(1)+4f\left(\tfrac{3}{2}\right)+f(2)\right]$$

$$=\frac{1}{6}\left[1\cdot\ln(1)+4\cdot\tfrac{3}{2}\ln\left(\tfrac{3}{2}\right)+2\ln(2)\right]$$

$$\approx 0.637$$

Trapezoidal rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a)+f(b)\right],\quad h=b-a$$

Midpoint rule:

$$\int_a^b f(x)dx \approx 2h\, f(x_0),\ h = \frac{b-a}{2}$$

$$x_0 = \frac{a+b}{2}$$

Ex/ $\displaystyle\int_0^2 f(x)\,dx$:

Trapezoidal rule: ≈ 5
Midpoint rule: ≈ 4
Simpson's rule ?

<u>Soln</u>

Trapezoidal rule: $5 = \frac{2}{2}\left[f(0) + f(2)\right]$
$f(0) + f(2) = 5$

Midpoint rule: $4 = 2f(1)$
$\Rightarrow f(1) = 2$

Simpson's rule: $\frac{1}{3}\left[f(0) + 4f(1) + f(2)\right]$

$= \frac{1}{3}\left[5 + 4(2)\right] = 13/3$

Composite quadrature

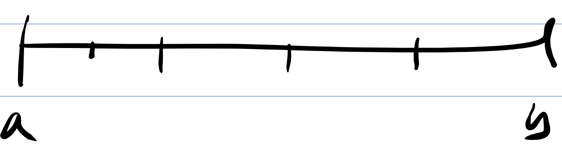
C.T.R. : error: $-\frac{h^3}{12}(b-a)f''(\mu)$,

$h=\frac{b-a}{n}$ $\qquad \mu\in(a,b)$

C.S.R. : error: $-\frac{1}{180}(b-a)h^4 f^{(4)}(\mu)$

$h=\frac{b-a}{n}$ $\qquad \mu\in(a,b)$

* n even



Ex/ Determine n required to approximate

$$\int_{-2}^{2} e^x\,dx$$

with an error tol. of 10^{-5} using:

a) C.T.R.

b) C.S.R.

Soln a) $\left|\frac{h^3}{12}(b-a)f''(\mu)\right|$

$$\max_{\mu\in[-2,2]}\left|\frac{(\frac{4}{n})^3}{12}(4)f''(\mu)\right|\le 10^{-5}$$

$$\frac{\left(\frac{4}{n}\right)^3 \cdot 4 \cdot e^2}{12} \leq 10^{-5}$$

$$n \geq 250.7 \rightarrow \boxed{n = 251}$$

b) $\left| \frac{1}{180} (b-a) h^4 f^{(4)}(\mu) \right|$

$$\max_{\mu \in [-2,2]} \left| \frac{1}{180} \cdot 4 \cdot \left(\frac{4}{n}\right)^4 f^{(4)}(\mu) \right| \leq 10^{-5}$$

$$\frac{4}{180} \cdot \left(\frac{4}{n}\right)^4 \cdot e^2 \leq 10^{-5}$$

$$n \geq 45.3 \rightarrow n = 46$$