Announcement: OH next week

Wed., 3:30-4:30pm

Open OH:

Tue., 7-8pm (even weeks?)

2 lands

1) Learning curve

2) Assessments

Differentiation

Def $f(x)$ diff.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Rules f, g diff.

Power rule: $h(x) = x^n$

$h'(x) = n x^{n-1}$

Chain rule: $h(x) = f(g(x))$

$h'(x) = f'(g(x)) g'(x)$

Product rule: $h(x) = f(x) g(x)$

$h'(x) = f'(x) g(x) + f(x) g'(x)$

Quotient rule: $h(x) = \frac{f(x)}{g(x)}$ $g(x) \neq 0$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

<u>Linear</u>: $h(x) = f(x) + g(x)$

$h'(x) = f'(x) + g'(x)$

c is a constant

$h(x) = c\, f(x)$

$h'(x) = c\, f'(x)$

$h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x)$

<u>Exponential</u> $h(x) = e^x$

$h'(x) = e^x$

<u>Ex</u> $f(x) = e^{x\sin x}$

chain rule

$$f'(x) = \underbrace{e^{x\sin x}}_{\text{exp}} \underbrace{[x\cos x + \sin x]}_{\text{product rule}}$$

chain rule

Ex/ $f(x) = \dfrac{e^x}{x+2}$ $x \neq -2$

$$f'(x) = \frac{(x+2)e^x - e^x}{(x+2)^2}$$

$$f(x) = e^x (x+2)^{-1}$$

$$f'(x) = e^x (x+2)^{-1} + e^x\left[-1(x+2)^{-2}\right]$$

Logarithms

Def $b > 0$

$$\log_b(b^x) = x$$

$$b^{\log_b(x)} = x$$

- b is the "base"
- inverse of exponentiation

Ex

x	$\log_2(x)$
1	0
2	1
4	2
8	3
16	4

$b = 2$

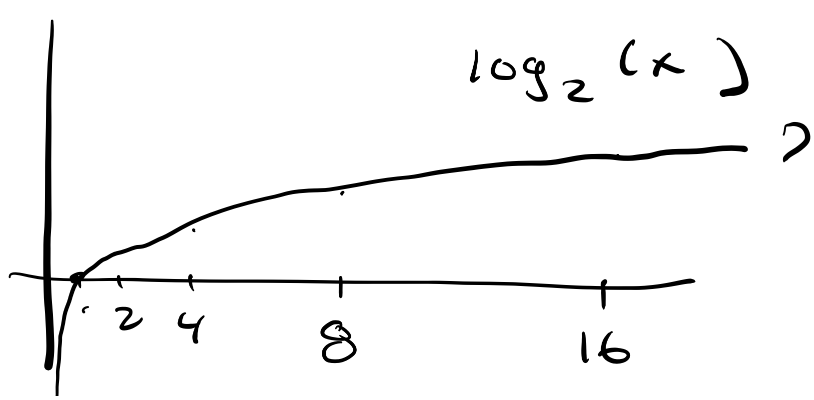
$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$



$\checkmark$

- $\log_2(x)$ is increasing
- asymptote at $x=0$

Inverse Functions & Derivatives

__Def__ If $f(x)$ is invertible on a given domain, the __inverse__ of $f(x)$ is $g(x)$ so that

$$g(f(x)) = x$$

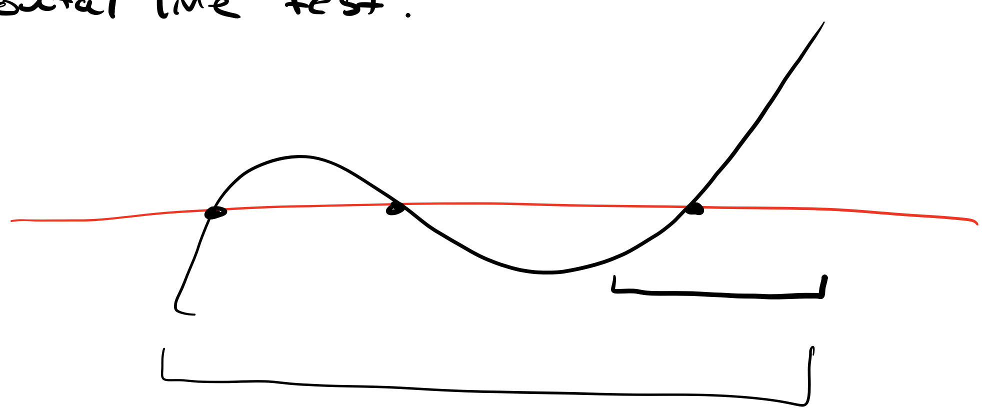
$$g(x) = f^{-1}(x) \leftarrow \underline{\text{inverse}}$$

When is $f(x)$ invertible?

__A__ f is one-to-one

if $f(x_1) = f(x_2)$, then $x_1 = x_2$

horizontal line test:

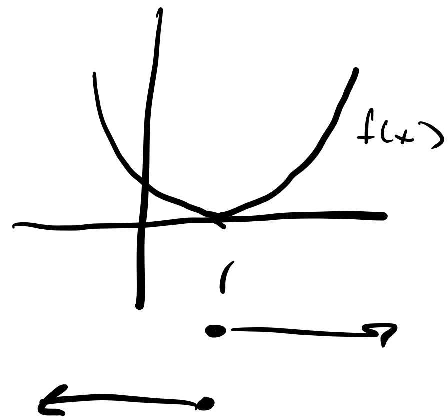


Ex/ $f(x) = (x-1)^2$

a) When is f invertible?

b) $f^{-1}(x)$?

Soln a) $x \geq 1$ ✓



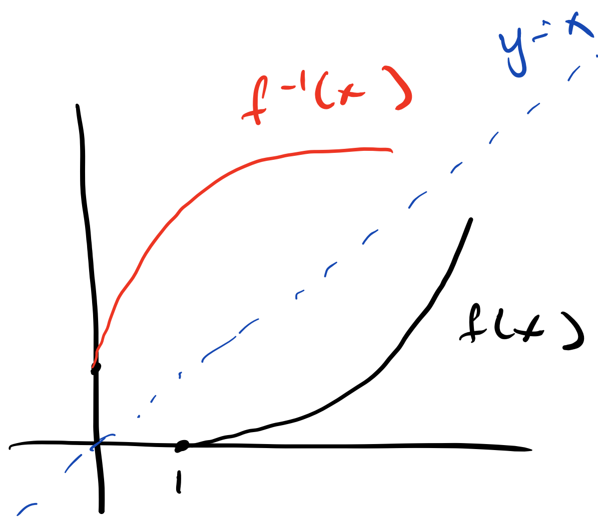
b) $y = (x-1)^2$

swap: $x = (y-1)^2$

solve: $\sqrt{x} = y-1$

$\Rightarrow y = \sqrt{x} + 1$

$f^{-1}(x) = \sqrt{x} + 1$



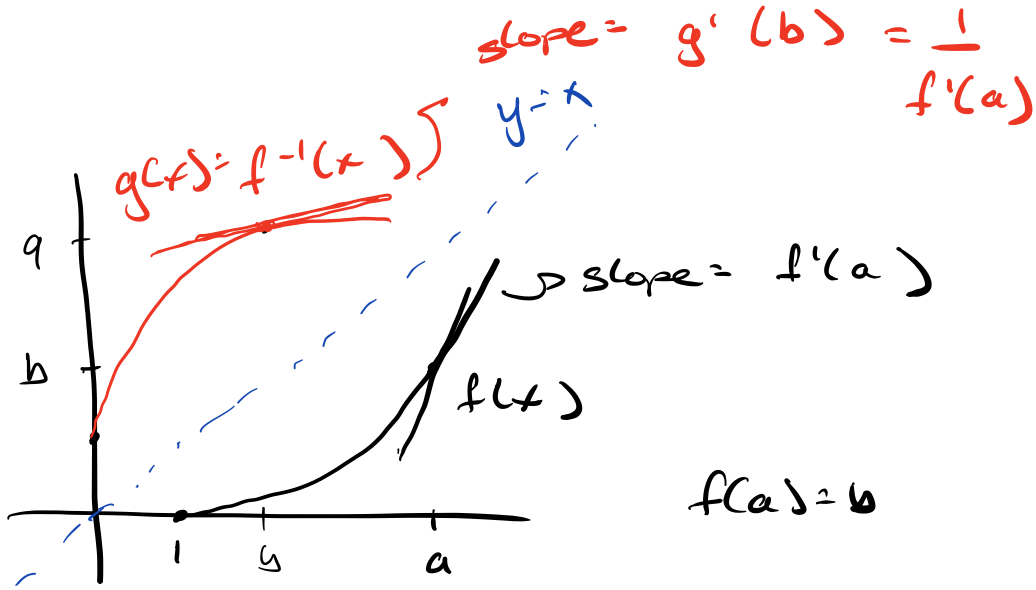
Derivatives of inverse functions

Fact $f(x)$ is invertible w/ inverse $g(x)$, $f'(a) \neq 0$, then

$$g'(f(a)) = \frac{1}{f'(a)}$$

Book: $f(a)=b \to g(b)=a$

$$g'(b)=\frac{1}{f'(g(b))}$$



$f(a)=b$

Ex/ $f(x)$ inv. $f^{-1}(x)=g(x)$

$f(3)=5$, $f'(3)=10$

$g'(5)$?

$g(5)=3$ $g(5)$

$$g'(5)=\frac{1}{f'(3)}$$ ✓

$$g'(5)=\frac{1}{10}$$

$$g'(f(a))=\frac{1}{f'(a)}$$

OR

$$g'(b)=\frac{1}{f'(g(b))}$$

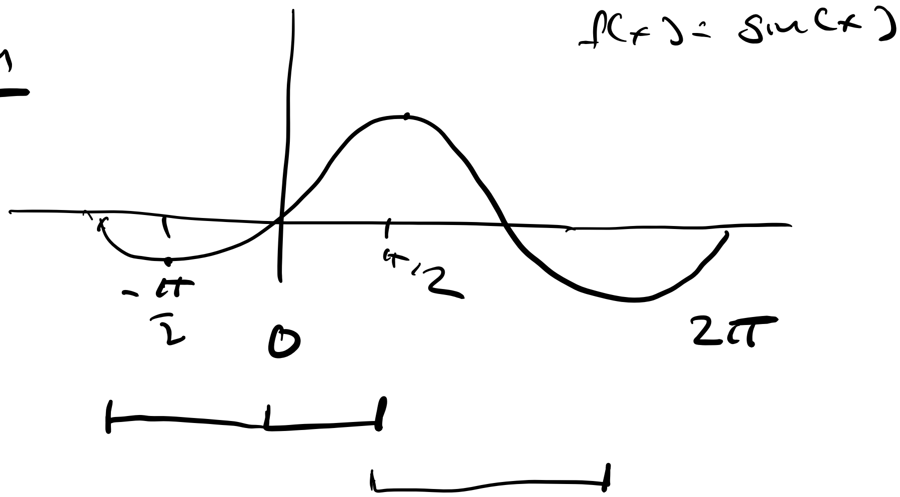
$f(a)=b$

Ex/ $f(x) = \sin(x)$

a) Choose a domain so that $f(x)$ is invertible

b) Call the inverse $g(x) = \arcsin(x)$
Find $g'(\frac{1}{2})$.

Soln



$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

b) $g'(f(a)) = \dfrac{1}{f'(a)}$

$g'(\frac{1}{2}) \Rightarrow f(a) = \frac{1}{2}$

$f(x) = \sin(x)$

a is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$a = \frac{\pi}{6} \Rightarrow f(\frac{\pi}{6}) = \frac{1}{2}$

$$f'(x) = \cos(x)$$

$$\rightarrow f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \neq 0$$

$$g'\left(f\left(\frac{\pi}{6}\right)\right) = g'\left(\frac{1}{2}\right)$$

$$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{2}{\sqrt{3}}$$