

Personal, Background, and Future Goals Statement

I first proved a mathematical statement for fun when I was sixteen, just because my math teacher, Mrs. Umekubo, threw out a challenge in an offhand comment. I will always remember the thrill of the proof and her subsequent mentoring during my senior thesis on statistical analysis of rainfall patterns. Her example and encouragement as a fellow woman of color inspired me to pursue further mathematical studies. Calvin University offered an opportunity to chase my dreams by awarding me a **Calvin Research Fellowship**, one of only twenty given to incoming freshmen. While at Calvin, I built a broad foundation by taking all the mathematics courses offered. When I decided to focus on applied mathematics, I gained computational experience through a minor in computer science. I also explored the application domain of chemistry, taking so many classes that I will earn an additional bachelor's degree.

My desire to pursue a Ph.D. in applied mathematics was cemented in January of 2019 when I attended the Joint Mathematics Meetings (JMM) to present my summer research. At JMM, I encountered fascinating connections between my previous research areas of algebraic geometry and optimization. This experience inspired me to pursue a career devoted to developing foundational analytic and computational tools for optimization.

Mathematics research is undeniably difficult. I've faced and overcome challenges such as unexpected counterexamples, insidious programming errors, and endless work with seemingly no progress. Nevertheless, I continue to study applied mathematics because I believe in its ability to solve seemingly intractable real-world problems. I hope to leverage scientific knowledge, mathematical insights, and computational tools for the benefit of society, also inspiring future mathematicians as Mrs. Umekubo inspired me.

Research: After finishing my first year of undergraduate study, I used my fellowship to engage in algebraic geometry research with Dr. James Turner of Calvin University's mathematics department. We sought to prove the n -variable Fröberg conjecture, which gives a formula for the Hilbert series of a specific class of quotient rings. Since the Moreno-Sociás conjecture implies the Fröberg conjecture, I worked on a proof of the Moreno-Sociás conjecture in n variables by characterizing the Gröbner bases of generic polynomial ideals. I taught myself the relevant material from algebraic geometry and the computer algebra systems Maple and Singular to compute 3-variable examples of Gröbner bases. Study of those cases led us to formulate a conjecture characterizing the Gröbner bases of generic ideals in n variables, which I then proved for the 2-variable case. I subsequently studied the 3-variable case and proved that the bases generated by our conjecture fulfilled a known condition for them to be Gröbner bases, up to a monomial multiplier. I wrote a C++ program to find this multiplier for all of the 3-variable cases I had computed, and I discovered conditions on those monomial multipliers, thus laying the foundation for a proof in 3 variables.

My work with Dr. Turner introduced me to the power of computation, so I then sought to apply computational mathematics in the natural sciences. This led to research in Calvin University's chemistry department with Dr. Douglas Vander Griend studying mathematical analysis of binding of host-guest molecules. I taught myself MATLAB to numerically simulate titration experiments of host-guest systems with known binding constants, experimental error types, and error magnitudes. I then wrote Monte Carlo simulations to determine the robustness of our lab's global mathematical analysis method. My studies showed that this multi-wavelength approach, as opposed to the single-wavelength method historically used in chemistry, improved the sensitivity of the mathematical analysis by at least three orders of magnitude. We then mathematically derived an optimal design for the titration experiment, and I developed numerical

studies to quantify the consequent gains in sensitivity. **We published this work in the *Journal of Chemometrics* with myself as second author [1].**

Given that traditional methods for analyzing host-guest reactions only work when the stoichiometries of the reactions are known, we extended our work to directly determine the stoichiometries of the reactions themselves. I proposed and co-wrote an object-oriented redesign of our group's software suite for estimating host-guest binding stoichiometries via nonlinear optimization algorithms. This redesign proved crucial as we tested a variety of algorithms and parametrizations. I studied the optimization literature to tune hyperparameters of the algorithms. I then evaluated the speed, convergence, and accuracy of several classes of optimization algorithms as applied to simultaneous determination of binding constants and stoichiometries. I simulated systems of up to thirteen reactions and determined that a batch of particle swarms followed by a multi-start Levenberg-Marquardt was the fastest and most accurate optimization approach. We are currently developing a supercomputer software package that leverages this methodology to perform high-throughput stoichiometry and binding constant estimation with bootstrapped error analysis. **We are preparing a manuscript for publication detailing our algorithmic methodology with myself as second author.**

My work with Dr. Vander Griend sparked a strong interest in optimization, numerical methods, and mathematical models. To gain additional experience in these areas of research, I applied and was accepted to a summer 2019 **NSF-funded REU** at **Cornell University** that focused on optimal control and dynamics. Under the direction of Dr. Andy Borum, I studied a dynamical model of highly flexible springs, which have exciting nanoscale applications in strain sensors, flexible electronics, and more. Flexible springs can exhibit unpredictable "jumps" and hysteresis in their configurations when their ends are perturbed. We sought to find stable and unstable configurations of these springs by defining and studying an optimal control problem. These equilibrium configurations minimize or maximize the spring's potential energy subject to the dynamics of its centerline, the positions of its endpoints, and a constraint prohibiting intersection of adjacent coils. I developed MATLAB programs to solve the problem via the shooting method and to visualize the corresponding spring configurations, subsequently studying springs with twisted and vertically extended ends. Working alone, I derived the equilibrium configurations of springs subjected to these forces, developed numerical methods to find unstable equilibrium configurations that cannot be found analytically, demonstrated the presence of saddle-node bifurcations in the solution space, and mathematically justified hysteresis in macroscopic springs. Since nanoscale springs have negligible weight, I initially studied a model without gravity, but am currently extending this project by considering the effect of gravity, changes to other physical parameters, and different forces on the endpoints. We plan to work with a collaborator to obtain experimental verification of my findings on the macroscale. **We are preparing a manuscript for publication based on my work.**

Intellectual Merit: I received recognition for my research ability via two nominations and an honorable mention for the **Barry Goldwater Scholarship** and a **second-author publication**, with two more manuscripts in preparation. As an undergraduate, I developed the problem-solving skills, knowledge, and drive to be a successful mathematician, and I gained a wide array of mathematical, computational, and practical experience in applying those skills to problems in other scientific disciplines. My undergraduate research prepares me well for my chosen field of optimization. I can study approaches to discrete optimization through algebraic geometry, computationally evaluate optimization algorithms arising in scientific contexts, and develop numerical methods for otherwise intractable optimal control problems. My research experience

also demonstrates that I have the perseverance and determination to do whatever it takes to solve problems, such as teaching myself the basics of MATLAB, algebraic geometry, and optimal control, each in just a few days. Because of my multifaceted background, I have the skillset, drive, and passion to leverage mathematics for the advancement of science and technology.

Broader Impacts: I have experienced firsthand the isolation felt by female mathematicians of color. Undergraduate classmates dismissed my interest in mathematics by attributing it to my mother's country of birth rather than my genuine, personal love for the subject. Every mistake I made intensified my anxiety that I could never be like the namesake mathematicians of the theorems I loved. And while I am extremely grateful for my many undergraduate research opportunities, none of my advisors have been women or people of color. These experiences fuel my passion for representation and inclusion. I want to be a successful mathematician and educator to mentor students with similar stories and be living proof that women of color belong both in mathematics specifically and in science more broadly.

I've already begun this mentoring work as a tutor and Girls Who Code facilitator. I saw that students from underrepresented groups felt an extreme amount of pressure to prove that they belonged in their fields of study. Not only did I help them understand the relevant coursework, I imparted conviction that they do, in fact, belong. As a graduate student, I will continue this work by mentoring undergraduate students, leading community outreach efforts, and advocating for women and people of color in mathematics.

True inclusion requires intentional community-building, a skill I gained during four years as a musician and worship planner for my college chapel. When I began working as a chapel planner, I noticed that we unintentionally excluded many international students by never singing songs in their native language. In my second week on the job, I consulted fluent speakers and wrote a multilingual version of a hymn refrain that incorporated Japanese, Dutch, and Swahili, three languages spoken on our campus that were not yet represented in our chapel events. I later led this refrain at an international Symposium on Worship, and several attendees asked me for the lyrics to bring back to their home congregations. I currently coordinate weekly chapels in languages other than English with people of diverse ages, disciplines, and cultures. Consequently, I have developed the ability to communicate across all kinds of barriers, a necessary skill for academic collaboration and promotion of diversity in scientific communities.

Of course, it takes more than good intentions to build a diverse, inclusive mathematics community. I attended a panel on mental health in the mathematics profession at JMM 2019 and heard many statements of support with few concrete plans for action and advocacy. Through my experience in planning, promoting, and running chapel events, I am well-prepared to spearhead practical efforts such as after-school science clubs at local high schools for underrepresented students, networking initiatives and events for women and people of color in the mathematics community, and advocacy in national organizations such as the American Mathematical Society.

Future Goals: I will pursue a Ph.D. in applied mathematics with the goal of becoming a research mathematician and professor. I want to advance our understanding of the world by developing mathematical and computational tools to model and solve complex scientific problems. I also seek to be an advocate and mentor for people who are told they do not belong in the scientific community. The NSF-GRFP will give me the resources and time to wholeheartedly pursue my education, research, and inclusive community-building efforts.

References: [1] Kazmierczak, N.P.; Chew, J.A.; Michmerhuizen, A.R.; Kim, S.E.; Drees, Z.D.; Rylaarsdam, A.; Thong, T.; Van Laar, L.; Vander Griend, D.A. Sensitivity Limits for Determining 1:1 Binding Constants from Spectrophotometric Titrations via Global Analysis. *Journal of Chemometrics*, 2019, 33, 3119.